NAG C Library Function Document

nag_zhptrd (f08gsc)

1 Purpose

nag_zhptrd (f08gsc) reduces a complex Hermitian matrix to tridiagonal form, using packed storage.

2 Specification

```
void nag_zhptrd (Nag_OrderType order, Nag_UploType uplo, Integer n, Complex ap[],
     double d[], double e[], Complex tau[], NagError *fail)
```

3 Description

nag_zhptrd (f08gsc) reduces a complex Hermitian matrix A, held in packed storage, to real symmetric tridiagonal form T by a unitary similarity transformation: $A = QTQ^H$.

The matrix Q is not formed explicitly but is represented as a product of $n-1$ elementary reflectors (see the f08 Chapter Introduction for details). Functions are provided to work with Q in this representation (see Section 8).

4 References

Golub G H and Van Loan C F (1996) *Matrix Computations* (3rd Edition) Johns Hopkins University Press, Baltimore

5 Parameters

1: **order** – Nag_OrderType *Input*

On entry: the **order** parameter specifies the two-dimensional storage scheme being used, i.e., row-major ordering or column-major ordering. C language defined storage is specified by **order** = **Nag_RowMajor**. See Section 2.2.1.4 of the Essential Introduction for a more detailed explanation of the use of this parameter.

Constraint: **order** = **Nag_RowMajor** or **Nag_ColMajor**.

2: **uplo** – Nag_UploType *Input*

On entry: indicates whether the upper or lower triangular part of A is stored as follows:

if **uplo** = **Nag_Upper**, the upper triangular part of A is stored;

if **uplo** = **Nag_Lower**, the lower triangular part of A is stored.

Constraint: **uplo** = **Nag_Upper** or **Nag_Lower**.

3: **n** – Integer *Input*

On entry: n, the order of the matrix A.

Constraint: $\mathbf{n} \geq 0$.

4: **ap**[dim] – Complex *Input/Output*

Note: the dimension, dim, of the array **ap** must be at least $\max(1, \mathbf{n} \times (\mathbf{n}+1)/2)$.

On entry: the Hermitian matrix A, packed by rows or columns. The storage of elements a_{ij} depends on the **order** and **uplo** parameters as follows:

if **order** = **Nag_ColMajor** and **uplo** = **Nag_Upper**,
a_{ij} is stored in **ap**$[(j-1) \times j/2 + i - 1]$, for $i \leq j$;

if **order** = **Nag_ColMajor** and **uplo** = **Nag_Lower**,
a_{ij} is stored in **ap**$[(2n-j) \times (j-1)/2 + i - 1]$, for $i \geq j$;

if **order** = **Nag_RowMajor** and **uplo** = **Nag_Upper**,
a_{ij} is stored in **ap**$[(2n-i) \times (i-1)/2 + j - 1]$, for $i \leq j$;

if **order** = **Nag_RowMajor** and **uplo** = **Nag_Lower**,
a_{ij} is stored in **ap**$[(i-1) \times i/2 + j - 1]$, for $i \geq j$.

On exit: A is overwritten by the tridiagonal matrix T and details of the unitary matrix Q.

5: **d**$[dim]$ – double *Output*

Note: the dimension, dim, of the array **d** must be at least $\max(1, \mathbf{n})$.

On exit: the diagonal elements of the tridiagonal matrix T.

6: **e**$[dim]$ – double *Output*

Note: the dimension, dim, of the array **e** must be at least $\max(1, \mathbf{n} - 1)$.

On exit: the off-diagonal elements of the tridiagonal matrix T.

7: **tau**$[dim]$ – Complex *Output*

Note: the dimension, dim, of the array **tau** must be at least $\max(1, \mathbf{n} - 1)$.

On exit: further details of the unitary matrix Q.

8: **fail** – NagError * *Output*

The NAG error parameter (see the Essential Introduction).

6 Error Indicators and Warnings

NE_INT

On entry, **n** = ⟨*value*⟩.
Constraint: **n** ≥ 0.

NE_ALLOC_FAIL

Memory allocation failed.

NE_BAD_PARAM

On entry, parameter ⟨*value*⟩ had an illegal value.

NE_INTERNAL_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

7 Accuracy

The computed tridiagonal matrix T is exactly similar to a nearby matrix $A + E$, where

$$\|E\|_2 \leq c(n)\epsilon\|A\|_2,$$

$c(n)$ is a modestly increasing function of n, and ϵ is the ***machine precision***.

The elements of T themselves may be sensitive to small perturbations in A or to rounding errors in the computation, but this does not affect the stability of the eigenvalues and eigenvectors.

8 Further Comments

The total number of real floating-point operations is approximately $\frac{16}{3}n^3$.

To form the unitary matrix Q this function may be followed by a call to nag_zupgtr (f08gtc):

```
nag_zupgtr (order,uplo,n,ap,tau,&q,pdq,&fail)
```

To apply Q to an n by p complex matrix C this function may be followed by a call to nag_zupmtr (f08guc). For example,

```
nag_zupmtr (order,Nag_LeftSide,uplo,Nag_NoTrans,n,p,ap,tau,&c,
pdc,&fail)
```

forms the matrix product QC.

The real analogue of this function is nag_dsptrd (f08gec).

9 Example

To reduce the matrix A to tridiagonal form, where

$$A = \begin{pmatrix} -2.28 + 0.00i & 1.78 - 2.03i & 2.26 + 0.10i & -0.12 + 2.53i \\ 1.78 + 2.03i & -1.12 + 0.00i & 0.01 + 0.43i & -1.07 + 0.86i \\ 2.26 - 0.10i & 0.01 - 0.43i & -0.37 + 0.00i & 2.31 - 0.92i \\ -0.12 - 2.53i & -1.07 - 0.86i & 2.31 + 0.92i & -0.73 + 0.00i \end{pmatrix},$$

using packed storage.

9.1 Program Text

```
/* nag_zhptrd (f08gsc) Example Program.
 *
 * Copyright 2001 Numerical Algorithms Group.
 *
 * Mark 7, 2001.
 */

#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagf08.h>

int main(void)
{
  /* Scalars */
  Integer  i, j, n, ap_len, d_len, e_len, tau_len;
  Integer  exit_status=0;
  NagError fail;
  Nag_UploType  uplo;
  Nag_OrderType order;
  /* Arrays */
  char    uplo_char[2];
  Complex *ap=0, *tau=0;
  double  *d=0, *e=0;

#ifdef NAG_COLUMN_MAJOR
#define A_UPPER(I,J) ap[J*(J-1)/2 + I - 1]
#define A_LOWER(I,J) ap[(2*n-J)*(J-1)/2 + I - 1]
  order = Nag_ColMajor;
#else
#define A_LOWER(I,J) ap[I*(I-1)/2 + J - 1]
#define A_UPPER(I,J) ap[(2*n-I)*(I-1)/2 + J - 1]
  order = Nag_RowMajor;
#endif

  INIT_FAIL(fail);
  Vprintf("f08gsc Example Program Results\n");
```

```
/* Skip heading in data file */
Vscanf("%*[^\n] ");
Vscanf("%ld%*[^\n] ", &n);
ap_len = n*(n+1)/2;
d_len = n;
e_len = n-1;
tau_len = n-1;

/* Allocate memory */
if ( !(ap = NAG_ALLOC(ap_len, Complex)) ||
     !(d = NAG_ALLOC(d_len, double)) ||
     !(e = NAG_ALLOC(e_len, double)) ||
     !(tau = NAG_ALLOC(tau_len, Complex)) )
  {
    Vprintf("Allocation failure\n");
    exit_status = -1;
    goto END;
  }

/* Read A from data file */
Vscanf(" ' %1s '%*[^\n] ", uplo_char);
if (*(unsigned char *)uplo_char == 'L')
  uplo = Nag_Lower;
else if (*(unsigned char *)uplo_char == 'U')
  uplo = Nag_Upper;
else
  {
    Vprintf("Unrecognised character for Nag_UploType type\n");
    exit_status = -1;
    goto END;
  }
if (uplo == Nag_Upper)
  {

    for (i = 1; i <= n; ++i)
      {
        for (j = i; j <= n; ++j)
          {
            Vscanf(" ( %lf , %lf ) ", &A_UPPER(i,j).re,
                   &A_UPPER(i,j).im);
          }
      }
    Vscanf("%*[^\n] ");
  }
else
  {
    for (i = 1; i <= n; ++i)
      {
        for (j = 1; j <= i; ++j)
          {
            Vscanf(" ( %lf , %lf ) ", &A_LOWER(i,j).re,
                   &A_LOWER(i,j).im);
          }
      }
    Vscanf("%*[^\n] ");
  }

/* Reduce A to tridiagonal form */
f08gsc(order, uplo, n, ap, d, e, tau, &fail);
if (fail.code != NE_NOERROR)
  {
    Vprintf("Error from f08gsc.\n%s\n", fail.message);
    exit_status = 1;
    goto END;
  }
/* Print tridiagonal form */
Vprintf("\nDiagonal\n");
for (i = 1; i <= n; ++i)
  Vprintf("%9.4f%s", d[i-1], i%8==0 ?"\n":" ");
Vprintf("\nOff-diagonal\n");
for (i = 1; i <= n - 1; ++i)
```

```
    Vprintf("%9.4f%s", e[i-1], i%8==0 ?"\n":" ");
  Vprintf("\n");
 END:
  if (ap) NAG_FREE(ap);
  if (d) NAG_FREE(d);
  if (e) NAG_FREE(e);
  if (tau) NAG_FREE(tau);

  return exit_status;
}
```

9.2 Program Data

```
f08gsc Example Program Data
  4                                                          :Value of N
  'U'                                                        :Value of UPLO
 (-2.28, 0.00) ( 1.78,-2.03) ( 2.26, 0.10) (-0.12, 2.53)
               (-1.12, 0.00) ( 0.01, 0.43) (-1.07, 0.86)
                             (-0.37, 0.00) ( 2.31,-0.92)
                                           (-0.73, 0.00)   :End of matrix A
```

9.3 Program Results

```
f08gsc Example Program Results

Diagonal
  -2.2800   -0.1285   -0.1666   -1.9249
Off-diagonal
  -4.3385   -2.0226   -1.8023
```